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ESSAY REVIEWS

Edited by KAREN HUNGER PARSHALL

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Convolutions in French Mathematics, 1800–1840. By Ivor Grattan-Guinness. Science Networks: Historical Studies, Vol. 2. Basel, Boston, Berlin (Birkhäuser). 1990. 3 Vols. 1602 pp. DM 398.

Reviewed by Thomas Archibald

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Historians, and historians of mathematics are no exception, have a fondness for the monumental, the encyclopedic. The desire to "do" a subject definitely, while perhaps chimerical, is often a strong one; and the products of such desires, however definitive, however tendentious, often perform the not-inconsiderable service of accumulating a wide body of diverse scholarship and assembling it in such a way as to focus discussion and serve as a basis for future work. Grattan-Guinness's *Convolutions* seems to me to share these features. While individual scholars will doubtless find much to debate in this work, several of its characteristics ensure that future historians of European mathematics in the period from 1800 to 1840 must consult the work and contend with its conclusions and allegations. Among these characteristics are the fact that it treats the essential developments in mathematical analysis, mechanics, and mathematical physics as parts of an interconnected whole, so that stylistic and substantive resemblances between work in the various areas are portrayed; furthermore, the fact that it attempts to take into account minor figures (at least, those who did important work) and integrate them into the historical picture in such a way that the influence of the principal figures can be more readily assessed; and, finally, the fact that the institutional setting, publication history, and trends in higher education are depicted as essential to the entire enterprise. None of these features is new in the history of mathematics, but

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to find all of them considered along with the content of the main mathematical memoirs is particularly important, and often very fascinating. Such a work might serve as a corrective to one-dimensional accounts which treat the content as though it had a life of its own; and to those which, following the mathematical fashion of a few years ago, treated applications as an unfortunate, parasitical outgrowth of pure mathematics, without which it cannot survive ("as an anteater cannot get along without ants," in Paul Halmos's phrase [1981, 287]). The very fact of Grattan-Guinness's book should eliminate this view among historians, and will with luck contribute to its demise among mathematicians as well.

Grattan-Guinness's central theme is the supplanting of the old guard, formed in the eighteenth century and at the head of the French mathematical sciences at its close, by a *nouvelle vague* of researchers issuing from the institutions created in the post-revolutionary period. The key figures at the turn of the century are, of course, Lagrange and Laplace, who with their hangers-on (especially Biot and Poisson) constitute the initial "insiders"; chairs in the grandes écoles, academy seats, positions on the Bureau des longitudes, and the like, were in large measure in their hands early in the period. The supplanters included Fourier, Fresnel, Cauchy, and Ampère, portrayed in *Convolutions* as outsiders to the Parisian establishment in one way or another, particularly during their early careers. The plot of the work consists in tracing the enfolding of these newcomers into the Parisian mainstream, which they eventually came to dominate; this process constitutes the "convolutions" (rather than revolution) of the title.

From the point of view of the mathematics involved, these developments are depicted as consisting of the simultaneous broadening of the various techniques of 18th-century calculus into limit-based mathematical analysis, and of mechanics into mathematical physics. They were accompanied by—even driven by—bitter personal rivalries which often translated into scientific terms, either in the form of disputes concerning methods (as in the case of the disagreements between Poisson and Cauchy concerning the foundations of calculus) or, in an institutional context, as debates over curriculum or in competitions for positions.

Grattan-Guinness presents these disputes with relish, both in their personal and in their scientific dimensions. One of the chief virtues of the work is the author's keen sensitivity to personal motivation, to the politics of scientific life, and to the many ways in which writers can express their allegiance. Veiled criticisms and symbolic acts were of great importance in this community. Consider, for example, Grattan-Guinness's comments on a remark of Francoeur's praising de Prony for maintaining the distinction between statics and dynamics:

> What can be the system receiving "happy simplification" from de Prony but that of [Lagrange] "reducing dynamics to statics" ... by the principle of virtual velocities? ... Was it to imitate the politics of his situation, as well as to honour another major figure, that Francoeur dedicated his book to Laplace? (p. 301)

Because of the competition for chairs and the importance of curriculum in waging the wars about appropriate methods, the relevant institutional history forms a vital part of the story, one which *Convolutions* handles well. Here the Ecole polytechnique is the most important institution, both because of the quality of its staff and students, and because its peculiar combination of rigorous mathematical training and an emphasis on application was so important for the shape of mathematics during the period. Another major contribution of the book is the sheer legwork that has gone into sorting out the chronology of developments. If this is often made trivial in later times by dated numbers of periodicals and the like, for the period at hand it is made vexedly difficult by publication delays, fictitious dates, private publications, unpublished printings, and the very diversity and restricted availability of the journal literature. In a similar vein, Grattan-Guinness has been extremely scrupulous in looking at the development of works (especially textbooks) that ran to several editions. A very considerable amount of unpublished material has also been consulted and used, and complete references are given. On all of these counts, for any of the subjects he discusses in detail, this work must serve as a fundamental resource for future researchers.

Despite the attention to context, the bulk of the work (in reading time if not in page count) consists of summaries of the mathematical content of individual treatises. These are assembled by subject, so that for the most part developments in the calculus are treated in separate chapters from those in mechanics and physics; and the treatment of each of these subjects is subdivided chronologically and by areas of specialization within the subject. The connections between the different parts of the treatise—which are essential if the plot is to be followed—are maintained by very extensive cross-references, which seem mostly reliable and are absolutely necessary if one seeks to trace the history of a given subject. Physically, *Convolutions* is divided into three volumes. The first contains "The Settings", and aims to provide both an overview of the *status quo* in calculus, mechanics, mathematical physics and engineering mathematics in 1800 and an account of the principal developments in these areas until roughly 1815 (though the work of Fourier, being outside the main Lagrangian/Laplacian lines of development, is not treated until the second volume). Volume 2, "The Turns," gives an account of the transformations undergone in these various areas between 1815 and 1840. In the realm of mathematical analysis, the work of Fourier and (especially) Cauchy forms the core of this section, while the studies of mechanics and mathematical physics examine major developments in optics, electricity and magnetism, thermal conduction, and elasticity. Volume 3, "The Data," contains selected passages from previously unpublished sources, biographical and chronological tables, the bibliography, and the indices.

Volume 3, which stands apart from the main exposition, requires separate comment, for it is fundamental to the rest of the work. It is divided as follows: 32 pages present 11 texts of selected manuscripts and printed works; 60 pages give tables which depict the biographies of the major figures, the people associated with the Ecole polytechnique, and an overall chronology; a bibliography, arranged alphabetico-chronologically, fills the next 129 pages; and finally, there are 57 pages of indexes, divided into three parts: persons, institutions and publications, and subjects.

The texts are fascinating. Most are transcriptions of manuscript materials, though in two instances they reproduce printed materials which did not achieve general distribution. Each text supports points raised in the main text, though many more such could also have been included. Perhaps at this point the economics of total page count did, in fact, enter the picture. Nevertheless, those included are generally interesting in their own right, depicting various aspects of the relations between the actors which are often omitted from other histories (and even, rather sadly, from recently published correspondence). Take, for example, the following delicious passage which occurs in a letter from Biot to Lacroix *circa* 1800:

> . . . voila donc le Bossutus ignarus hominusque membre du jury des ecoles centrales. voila donc Lagrange et Laplace qui, devenus conservateurs abandonnent le gouvernement des Sciences pour les remettre en des mains ineptes et debiles. . . . je suis sûr que Bossutus aimerait mieux renoncer pour toute sa vie à faire des mathematiques, qu'à prendre du tabac. (p. 1321). [The unorthodox spelling and punctuation are as in the original.]

Not everything contained here is merely gossip, however, though the gossip is worth the price of admission. We also find passages of scientific importance (Fourier's recognition of the fallibility of Newton's law of cooling), teaching evaluations (of Cauchy, by de Prony, unflattering), and plans for curriculum change (at the Ecole polytechnique, by Coriolis). Such materials are of great value to historians of many stripes, and we may hope that their use and inclusion here will stimulate a trend.

This is true likewise of at least some of the tables. The biographical table for the major figures will be of considerable use to anyone writing in the field, compressing as it does a great deal of information which is not readily available in any other source. For example, Poggendorff's information about the French is generally spotty. Of particular interest is the column on the main known *Nachlass* of many individuals. Likewise the chronological summary is laid out so that inspection permits one to extract events by subject without too much difficulty, and contains references to points in the work where the event in question is discussed. Failing the inclusion of a diskette with such information in a database format, these tables will be of considerable assistance to future students of these works.

The bibliography is undifferentiated; for example, there is no separation between primary materials and historical commentaries, or on the basis of subject. Probably this would have been quite difficult to do in a meaningful way. It is not an easy bibliography to browse in, but this is compensated for by the fact that the authors of secondary works are included in the index of persons, so that, for example, on finding a bibliographical reference to Elie Cartan we can rapidly find that the work has been referred to in the discussion of the geometry of complex numbers. A useful feature of the bibliography is the inclusion at the beginning of a list of the principal journals of the primary literature, some 50 of them. Somewhat annoying to this reader was the omission of publishers from the descriptions of books. The fact that there is a separate index of institutions and publications reflects a major virtue of the work remarked on earlier, namely, that the history of the publications is taken very seriously as a component of the development of the field.

The story itself, as told in the first two volumes, far surpasses in detail any synthetic work previously attempted. Thus, the discussion of Lagrangian calculus and its influence up to 1815 occupies almost 150 pages (as compared, for example, to a dozen in Bottazzini [1986]), and is further supplemented by an account of the influence of the *Méchanique analitique*. This extended treatment permits an examination of the effects of Lagrange's algebraic version of analysis on his own researches in a variety of areas (such as differential equations and the calculus of variations), an examination which has previously been undertaken in a scattering of secondary works of varying availability. Similarly, Grattan-Guinness recounts the work of individuals who usually have figured only as names in earlier surveys (such as Servois and the Français brothers). Likewise the treatment of Cauchy's analysis in the second volume presents not only many details of Cauchy's own work that have received only fleeting attention from previous writers, but also shows the reaction to this work of Poisson and others. These and other accounts have benefitted significantly from earlier secondary work, a good deal of which is from the nineteenth century and now difficult of access. Not everyone will agree with all of Grattan-Guinness's assessments of earlier work; and in some cases (such as the chapter on optics) I felt that more effective use could have been made of existing secondary treatments without seriously compromising the independence and originality of the account in *Convolutions*. The desire to be comprehensive occasionally does obscure the main lines of the development, somewhat inevitably, as the author warns at the beginning. This desire sometimes leads to a rather breathless effect, as well, especially in the material toward the end of the second volume on the developments of the 1830s.

An enormous amount of information is contained in these 1600 pages, then. But one must ask whether the various areas are accurately and comprehensibly portrayed; whether the material selected for detailed treatment is well chosen; how well the links are made between the different parts of the story; and whether the organizing, explanatory principles and the conclusions drawn using them are reliable. On all of these counts the work is a mixed success, and though I concentrate in what follows on problems with the work, I feel that the extent to which these detract from the very real contribution of these volumes will depend on the reader and his or her predilections.

The very bulk of the work is itself a problem. The work was not well served by its editors in that some of the less essential part of that bulk could have been eliminated, while other portions needed expansion. In the former class is a lot of structural prose (of the variety "having done A, we shall now do B"), excessive reference to material which will be covered below or has been covered above (though the extensive cross-referencing is very well done), and repetition of casual remarks (such as the fact that Condorcet's interest in signs was a form of semiotics). A certain number of comments could also have been dispensed with. For example, do we really need to be exhorted at the beginning of a 23-page table of contents that we should study it in more detail than might normally be considered? Headings do not always correspond well to what lies beneath them, as, for example, in Section 14.2.2, "Ampère's adoption of the electrical aether, 1820–21," in which aether is not mentioned. The apparatus, generally good, occasionally founders or disappoints. For example, the frequently used abbreviation *PV* is not in the list of abbreviations on p. 73 and on the back end-papers; it is explained on p. 84 (after its first use) to refer to the *Procès-verbaux* of the mathematical and physical class of the Institut for the years from 1795 to 1835 (which were published only in this century). These and other such difficulties should have been addressed by a careful reader prior to publication.

As regards methods, one of the organizing or explanatory principles cherished by Grattan-Guinness is the notion of *Denkweisen,* or styles of reasoning in mathematics. These he divides into the algebraic, the geometric, and the analytic, and frequent reference is made to the adherence of one mathematician or another to such a style (a summary table is included near the end of Volume 2). The notion, though occasionally helpful (as, for example, in the case of Lagrange, an avowed opponent of the use of diagrams in mechanics and a proponent of a theory of calculus based on algebra rather than on limits), is usually left vague. Yet it is frequently invoked to establish a kind of "school"-like filiation, sometimes in the absence of specific references to work or detailed employment of like methods. I was left with the impression that much needs yet to be done in order to sort out the relative roles of geometry, algebra, and limits in the foundations and the practice of the calculus during the period in question, and with considerable doubt about the usefulness of these *Denkweisen* even as organizing principles.

There are some features of the style of the work which readers may find off-putting. Some of these are simply quirky, such as the use of τ to denote half the area of the unit circle (on the grounds that it has one "leg", while pi has 2). Another is the insistence (applied a little inconsistently) on adherence to original notation, and of noting departures from it. While it is admirable not to introduce anachronisms via notation, I do not see the value of noting, as Grattan-Guinness does on p. 304 when reproducing a diagram of Fourier's, that the lettering has been changed from the original (especially since the reason seems to be that the original lettering was more confusing). This adherence to the original way of writing the mathematics is all the more remarkable since, with very few exceptions, French originals are quoted in English translation. Presumably in keeping with the idea of not violating the original, the translations seem very literal, sometimes awkwardly so ("... according as if it is the first ...," p. 137; "... this is why that metaphysics is almost always ...," p. 139). Yet even so they are readily comprehensible, while some of the mathematics is quite hard going even where one is familiar with the area and the historical notation. I found myself wishing more than once that work being expounded had been "translated" into more compact notation, especially in the physical portions of the treatise, with remarks concerning the significance of the difference in notation. Here I mention in particular the lengthy account of Ampère's work. Ampère employed a left-handed Cartesian coordinate system to calculate results that are now almost always discussed using either vector methods or differential forms. By adhering to the original notation, Grattan-Guinness ensures that the commentary he produces is no more readily accessible to the understanding than the original. Surely part of the point of writing the history is to allow the reader to assess the contribution of the figure in question; this is rendered very difficult when the lines of thought are obscured by dreadful historical notation.

This brings us to the question of the overall quality of the account of mathematical developments. In my view, the work chosen for exposition has been chosen well. Synoptic treatment is given of many works of generally acknowledged importance, though, of course, at varying depths. The biggest single problem with the work is that these synopses, valuable as they may be in conjunction with the original, are frequently too concise for clarity. Thus, we are often called upon to accept an account of a mathematician's line of thought without having really adequate evidence before us; and stimulating as the author's remarks often are, I regularly found myself wishing that I could share the information which had led him to the conclusion in question. In this regard the material on calculus and analysis seemed to me the best, though even here I was often greedy for detail. When I did seek greater detail—for example, by consulting Lagrange's works concerning his efforts to found calculus on power series—I found the account in Grattan-Guinness comprehensible and reliable given the level of detail it attempts. Thus, while *Convolutions* guides us to the literature, and may even guide us through it to some extent, it does not serve (at least, not for this reader) as a free-standing account of the matters at hand. The accounts of work in mathematical physics, which usually omit detailed discussion of experimental work as the companion of analytical developments, I found deeply incomprehensible in spots. I realize that these accounts are intended as a corrective to the usual neglect of mathematics in historical treatments of physics, and the author does succeed in drawing to our attention much important mathematics. However, if, with the author, we accept the fact that mathematics and physics were inextricably linked at the time, we simply must see more of the physical thought to know what is going on. My confidence was not enhanced by the occasional error of the author, for example in the section on optics, where the velocity of a particle vibrating transversally in the medium is confounded with the velocity of the wave front (p. 863).

As for the physical production of the volumes: I noticed a good many typographical errors, though these are not too confusing on the whole. A page correcting some of them occurs at the end of Volume 3. The binding appears to be of good quality. The paper is very soft, and flecked with impurities; I have some doubts about its durability, but a lack of durability (due to acid content or sizing) does not distinguish it from many of the coated papers unfortunately used in many other mathematical works today. One may regret, however, that better paper was not used in a work which is likely to remain useful for a long time.

I conclude by stressing the very considerable achievements of these volumes, and their usefulness to future researchers. If the frequently attempted task of depicting the details of the mathematics is not entirely successful, the overall picture of the developments is convincing and interesting, and the very establishment of the chronicle is of fundamental importance. A full elaboration of the picture drafted here will take many writers many years. Characteristically, Grattan-Guinness concludes the main text with a page in which he expresses his hope for the impact of these volumes. As he states, "the potential uses of this study are considerable" (p. 1304), and I feel certain that the historical analysis which he has substantially furthered will bear fruit.

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Reviewed by Victor J. Katz

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The massive 1238-page history of mathematics by Morris Kline, *Mathematical Thought from Ancient to Modern Times,* begins with the statement, "Mathematics as an organized, independent, and reasoned discipline did not exist before the classical Greeks of the period from 600 to 300 B.C. entered upon the scene" [Kline 1972]. Spending then 12 pages on Babylonian mathematics and nine pages on Egyptian mathematics, Kline proceeds to deal with the Greeks for 159 pages. After a 17-page chapter on "The Mathematics of the Hindus and Arabs," he spends the remainder of the book on the mathematics of Europe and the United States. The two books under review challenge this picture of the absolute centrality of Europe to the development of mathematics.

Marcia Ascher and George Joseph deal with the mathematics outside of Europe, however, in entirely different ways. Joseph's book is fairly traditional in approach. He considers the documented evidence of mathematical thought in Egypt and Babylonia and then proceeds to a detailed treatment of the mathematics of China, India, and the Islamic world, a treatment virtually entirely lacking in Kline's tome. Ascher, however, considers the mathematical ideas of people in traditional, or "small scale" cultures, cultures in general without a written tradition. Thus, she must bring in evidence from anthropology and ethnography to buttress her case